Shock-Turbulence Interaction

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Abstract. For the general purpose of investigating phenomenon of shock-turbulence interaction, we consider here the problem of a plane shock wave propagating in a turbulent flow field. We compute this by the kinetic model approach with use of the Boltzmann equation in the BGK approximation. We produce a plane shock wave by a shock tube flow type computation and run this into a prefabricated turbulent flow. We observe the shock thickness widening and compare it with the one by a mixture length theory as well as data of an experimental study in [4].

INTRODUCTION

For the general purpose of investigating phenomenon of shock-turbulence interaction [1], we consider here the problem of a plane shock wave propagating in a turbulent flow field. We compute this by the molecular kinetic model theory approach of the Boltzmann equation with the BGK approximation [2]. In practice, we prepare first a steady plane shock wave in uniform gas by a shock tube flow type computation, and then run this into a prefabricated two-dimensional isotropic turbulent field [2]. Among results, particular attention is paid to the widening of the shock front thickness due to the interaction with the turbulent flow. It is compared also with the shock thickness given by a mixing length theory [3] and data by a corresponding experiment [4].

BOLTZMANN−BGK EQUATION IN INTEGRAL FORM

We consider two-dimensional flow field and use the Boltzmann – BGK equation in integral form [2] as

$$f(\xi, \mathbf{x}+\xi\Delta t, t+\Delta t) - f(\xi, \mathbf{x}, t) = \nu(f_0 - f) \tag{1}$$

, where $f = f(\xi, \mathbf{x}, t)$ is the molecular distribution function, $\xi = (\xi_x, \xi_y, \xi_z)$ is the molecular velocity, $\mathbf{x} = (x, y)$ is the spatial coordinate. $\nu = \nu(\mathbf{x}, t)$ is the collision frequency and $f_0 = f_0(\xi, \mathbf{x}, t)$ is the local Maxwellian:

$$f_0 = N(\pi T)^{-3/2} \exp\{-C^2/T\}, \quad \mathbf{C} = \xi - \mathbf{u} \tag{2}$$

with the number density N, the temperature T and flow velocity $\mathbf{u}$ which are expressed in non-dimensional form based on a basic length L and the referencing number density N_0 and temperature T_0. Further we assume the Maxwell molecular model to have $\nu = k_0 / K_n.N$, $Kn = l_0 / L$, $k_0 = 8/5\sqrt{\pi}$, where Kn, l_0 represent Knudsen number and mean free path respectively.
For the convenience of numerical computation of two-dimensional flow, we use the reduced distribution functions

g, h by $(g,h) = \int (1, \xi_z^2) f d\xi_z$ and their local Maxwellians g_0, h_0 by

$$g_0 = (N/\pi T)\exp\{-(C_x^2 + C_y^2)/T\}, h_0 = Tg_0/2 \tag{3}$$

So that we can reduce the independent variables to (ξ_x, ξ_y, x, y, t).

PLANE SHOCK WAVE IN SHOCK TUBE TYPE COMPUTATION

We produce first a steady plane shock wave in uniform gas by a shock tube flow type computation. For this, we postulate a shock tube like configuration as shown in Fig 1, where two sections separated by the membrane filled with gases of different pressures p_1, p_2. Computations are performed with use of the kinetic model equation above for the sudden removed of the separating membrane. One of the results is shown in Fig 2 in pressure distributions at different times where $p_1/p_2 = 5$.
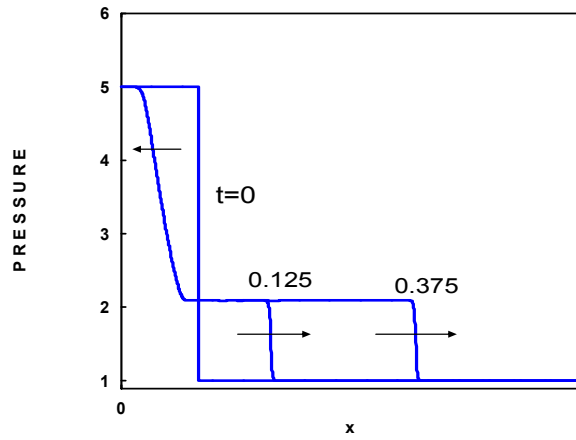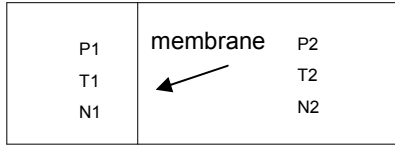

FIGURE 1. Shock tube type configuration. **FIGURE 2.** Plane shock wave profile propagating in shock tube type computation.

TWO-DIMENSIONAL ISOTROPIC TURBULENT FLOW FIELD

We produce a two-dimensional isotropic turbulent flow[2], that is space periodic in a square region. A random initial condition is set at time t=0, as a local Maxwellian f_{00} is set with their flow velocity $\mathbf{u}_{00}$, number density $N_{00} = 1$ and temperature $T_{00} = 1$ for $\mathbf{x} = (x, y)$ in a unit squire region

$$f_0 = N_{00}(\pi T)^{-3/2}\exp\{-C^2/T\} \quad , \quad \mathbf{C} = \boldsymbol{\xi} - \mathbf{u}_{00} , \tag{4}$$

where $\mathbf{u}_{00}$ obeys the isotropic energy spectrum $E(k)$：

$$E(k) = 2\pi \left\langle \frac{1}{2}\mathbf{u}_{00}(\mathbf{k}) \cdot \mathbf{u}_{00}(-\mathbf{k}) \right\rangle = K^2 k^2 \exp\{-(k/k_0)^3\} \tag{5}$$

where $\mathbf{u}_{00}(\mathbf{k})$ is the Fourier transform of $\mathbf{u}_{00}$ defined as

$$\mathbf{u}_{00}(\mathbf{k}) = \int \mathbf{u}_{00}(\mathbf{x})\exp(i\mathbf{k} \cdot \mathbf{x})d\mathbf{x} . \tag{6}$$

where K is an adjustable constant for the normalization, and k_0 (the wave number magnitude of energy containing eddies at the initial state) is chosen as $k_0 = 10$. The individual mode is given randomly as follows:

Let

$$\mathbf{k} = 2\pi\mathbf{n}, \quad \mathbf{n} = (n_1, n_2), \quad n_i = 0, 1, 2, --- \quad \text{for } i = 1, 2$$

and set

$$\mathbf{u}_{00} = (u_{00}, v_{00})$$

with

$$u_{00} = \sum_{\mathbf{n}} a_{\mathbf{n}}(n_1, n_2) \sin 2\pi\{n_1 x + n_2 y + \varepsilon_{\mathbf{n}}(\mathbf{n})\}; \quad a_{\mathbf{n}} = \sqrt{E/k^2}, \tag{7}$$

and similar expressions for v_{00}, where $\varepsilon_{\mathbf{n}}(\mathbf{n})$ are random numbers in $[0,1]$.

Computation is performed for $K_n = 0.001$, time step $\Delta t = 0.0001$, 100×100 divisions for $\mathbf{x}$ and $\Delta\xi_x = \Delta\xi_x = 0.5$ for $-5 < \xi_x, \xi_x < 5$ in $\mathbf{x}$. The initial Reynolds number is $R_e = 6700$ with $M = 4.5$ of the maximum value of the initial velocity $U = 62K\xi_0$ and $K = 0.06$. One example of results obtained is shown in Fig.3 for the density contours.
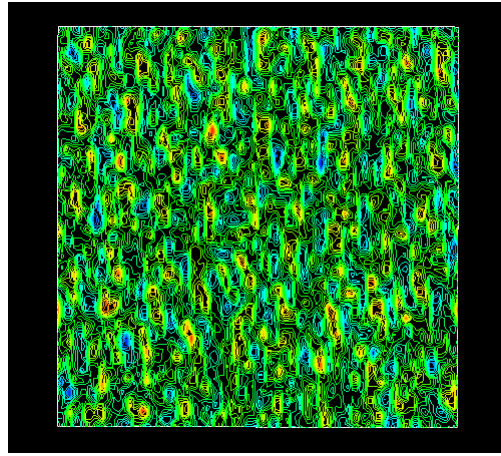

FIGURE 3. Density contours of the two-dimensional turbulent flow field produced numerically by a random initial condition having a proper energy spectrum.

SHOCK WAVE IN TURBULENT FLOW FIELD

We put the shock wave produced by the shock tube type computation as above into the turbulent flow as prefabricated as shown above in Fig.3. In practice, we performed in the two ways (i),(ii) :

（ i ）The shock wave is pushed into the field from its left end.
（ ii ）Place the wave standing in the flow field

Examples of pressure distribution results are shown in Fig.4 and Fig.5 respectively for the cases (i) and (ii). A particular attention is paid to the widening of the shock front thickness (TH) due to the interaction with the turbulent flow, which is compared with the one seen in Fig.2. The shock thickness TH seen in Fig.5 is about 0.13 to the present dimensionless scale.

The widening of the shock thickness due to turbulence is expected to be caused by the Reynolds stress and it is naturally related to the scale of the mixing length in the turbulent flow field. The thickness given by a mixing length theory [3] is given as

$$TH = \frac{k}{C_0} \frac{M}{M^2 - 1} \cdot 2\log_e 9 \qquad k = -\overline{u'^2} \left/ \frac{d\bar{u}}{dx} \right. \tag{8}$$

,where C_0, k ,M, $\overline{u}$ represent the velocity of sound, eddy kinematic viscosity, Mach number of shock wave and time average of the velocity, respectively. The TH value by eq (8) to the above example case is comparable size of 0.27 to 0.13 above.
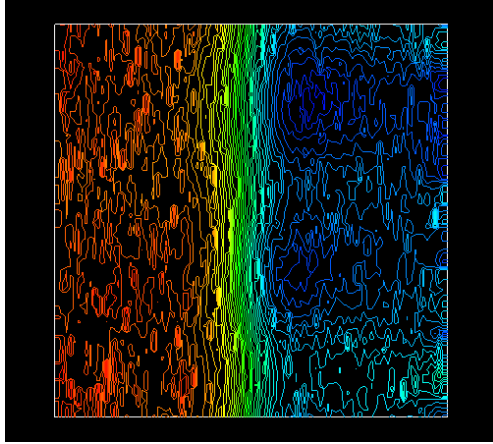

FIGURE 4. Plane shock wave pushed into the turbulent flow field from its left end
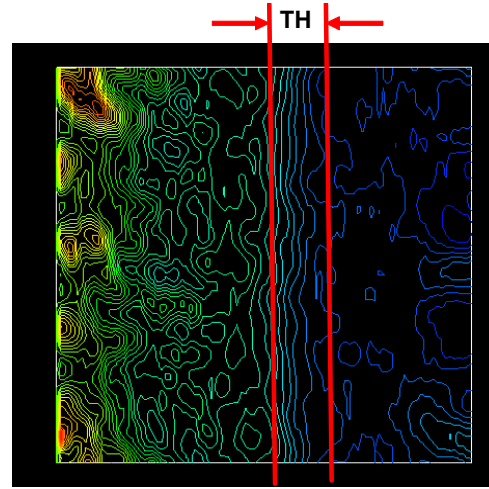
FIGURE 5 Plane shock wave stationary in turbulent flow showing its thickness.

The result is compared also with data by a corresponding experiment [4] in which the interaction between a shock wave produced by a shock tube with atmospheric grid turbulence in a low speed wind tunnel is examined. Its pressure data at various points in the tunnel is reproduced in Fig.6(a) and it is compared with the temporal pressure data at x=0.5,y=0.5 of the present study in Fig.6(b). Notice the same kind of pressure value fluctuations appearing in both cases.
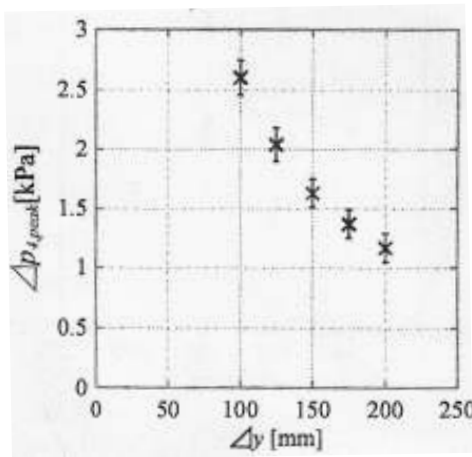
(a)
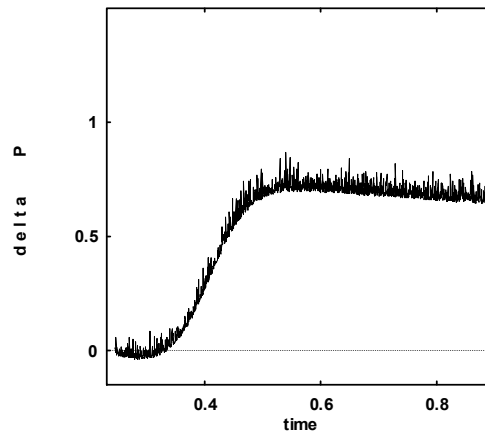
(b)

FIGURE 6. (a) Reproduction of pressure data at various points in wind tunnel experiment in [4]. (b) Present numerical temporal pressure data at x=0.5, y=0.5

SUMMARY

Considered the problem of a plane shock wave propagating in a turbulent flow field.

Computed the flow field by the molecular kinetic model approach of the Boltzmann equation with the BGK approximation theory.

Ran a plane shock wave produced in a shock tube flow type computation into a prefabricated two dimensional turbulent flow.

Among results, a particular attention was paid to the widening of shock front thickness due to its interaction with the turbulent flow, which is compared with the one in the original shock wave. Compared also with the shock thickness given by a mixing length theory[3], and data by a corresponding experiment[4].

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